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GENERALIZED CHOW TESTS FOR STRUCTURAL CHANGE: A COORDINATE-FREE APPROACH*

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1. INTRODUCTION

An important way of assessing the reliability of an econometric model, especially in view of making forecasts or policy simulations, consists in checking whether it is stable over time (see Lucas [1976]). Frequently, this problem can be formalized as one of testing whether the coefficient vectors in several regressions (corresponding to disjoint subperiods) are equal. Namely, one considers a set of m regressions:

$$(1.1) \qquad \underline{y}_i = X_i \underline{\beta}_i + \underline{u}_i, \qquad i = 1,\ldots, m,$$

where $\underline{y}_i$ is a $n_i \times 1$ vector of observations on a dependent variable, X_i is a $n_i \times k$ nonstochastic matrix of explanatory variables having rank r_i, $\underline{\beta}_i$ is a $k \times 1$ vector of coefficients and $\underline{u}_i$ a $n_i \times 1$ vector of random disturbances ($i = 1,\ldots, m$). It is also assumed that $(\underline{u}'_1,\ldots, \underline{u}'_n)' \sim N[\underline{0}, \sigma^2 I_n]$, where $n = \sum_{i=1}^{m} n_i$. The null hypothesis to be tested is H_0: $\underline{\beta}_1 = \cdots = \underline{\beta}_m$.

To the extent of our knowledge, the above problem has never been considered in its full generality in the literature. For cases where each regression has sufficient sample size to allow a separate estimation of β_i and $\sigma^2 (r_i = k < n_i, i = 1,\ldots, m)$, the solution is a standard analysis-of-covariance test given by Kullback and Rosenblatt [1957]. However, these authors did not deal with the frequent case where one or several of the subperiods have an insufficient sample size ($n_i < k$). In view of such situations, Chow [1960] considered the case of two samples ($m = 2$), one of which is undersized ($r_1 = k < n_1$ and $r_2 = n_2 \leq k$), and proposed to use a predictive test comparing the vector $\underline{y}_2$ with the vector of predictions $X_2 \hat{\underline{\beta}}_1$ based on the regression from the other sample. This test as well as the analysis-of-covariance test for $m = 2$ (which Chow also derives) are generally known to econometricians as the "Chow tests" for structural change (a terminology we shall retain here). Further discussions of these tests were provided by Fisher [1970], Gujarati [1970], Harvey [1976], Andersen [1977], Rea [1978], Wilson [1978] and Dufour [1980].[1] An important disadvantage of the line of reasoning

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[1] The same problem, when disturbance variances in the two regressions are unequal, was considered by Toyoda [1974], Schmidt and Sickles [1977], Jayatissa [1977], Goldfeld and Quandt [1978] and Watt [1979]; in this paper, we will not consider this type of situation. Corresponding Bayesian posterior odds (for the case $r_i = k < n_i$, $i = 1,2$) were presented by Zellner and Siow [1979]. Note also that the problem of testing H_0 in the context of "seemingly unrelated regressions" (for $n_1 = \cdots = n_m$ and $r_i = k$, $i = 1,\ldots, m$) was considered by Zellner [1962].

adopted by Chow [1960] in order to deal with problems of undersized samples is that it cannot be applied to cases in which the number of periods exceeds 2. For example, if we have one undersized sample with two or more samples of sufficient size, which sample (or combination of samples) should be used to generate the predictions of the undersized sample — if one does not wish to arbitrarily reduce the problem to the two sample case by (questionably) assuming the $m-1$ other regressions have the same coefficients? Similarly, if we have several undersized samples with several samples of sufficient size, it is even less clear how the predictions should be made. Furthermore, even if the union of the undersized samples has sufficient size (so that these could be merged and the analysis-of-covariance test be applied), merging the undersized samples is not always appropriate, as it implies that the coefficient vector is the same among these samples, a hypothesis we may precisely want to question.

The first purpose of this paper is to provide explicit and easily applicable solutions to such problems of undersized samples, thereby generalizing the predictive Chow test. In order to do this, we shall first derive a general solution to the problem of testing the equality of coefficient vectors in several regressions when explanatory variable matrices have arbitrary ranks ($r_i \leq k$, $i=1,\dots, m$). Problems of undersized samples can then be dealt with in a simple manner, because they are a special case of the broader problem. Besides, it is worthwhile noting that this more general result has further practical applications: in particular, situations with all or some $r_i<k$ can occur easily when dummy variables are present among the regressors (e.g., if a dummy variable remains constant over a subperiod); furthermore, this general set-up has the additional flexibility of allowing the exclusion of certain variables from some of the regressions (by putting zeros in the appropriate columns)[2].

A natural extension of the above class of problems consists in testing equality between subsets of coefficients in m regressions. This problem was also considered by Kullback and Rosenblatt [1957] and Chow [1960], though under the same restrictive conditions described previously. Accordingly, the second purpose of this paper will be to extend these results in various ways, allowing, in particular, for explanatory-variable matrices with arbitrary ranks (hence, again, for a range of situations with undersized samples wider than previously considered by Chow) as well as for different numbers of coefficients among the m regressions.

Third, it appears that, in order to deal with these non-full rank problems, an algebraic treatment like the one of Chow [1960], or even the neater exposition given by Fisher [1970], would have been quite burdensome. We used instead a geometric, or coordinate-free, approach (see Herr [1980], Kruskal [1961, 1968]),

[2] Of course, in such a case, the null hypothesis really being tested states equality restrictions only between those coefficients affecting non-zero variables; i.e., H_0 and this (wider) null hypothesis are observationally equivalent. More generally, the main consequence of having $r_i<k$ for some i, is that there will be alternatives which will be observationally indistinguishable from H_0, the same type of "indeterminacy" problem pointed out by Rea [1978] for the Chow test with an insufficient number of observations.

based on the elegant geometric interpretation of likelihood ratio tests in the classical linear model given by Scheffé [1959]. Although this approach has been discussed by Malinvaud [1969, Ch. 5], it seems it has been very little used in the econometric literature. Consequently, a third purpose of the paper will be to provide an illustration of the simplicity as well as the fruitfulness of geometric reasoning in the analysis of linear models.

The general likelihood ratio test against H_0 is presented in Section 2. In Section 3, the Kullback-Rosenblatt and Chow tests are shown to be special cases of the general test obtained, and the explicit test for the important case where one or several regressions among $m \geq 3$ regressions have insufficient numbers of observations is given. The problem of testing equality between subsets of coefficients in m regressions is considered in Section 4. Finally, in Section 5, one of the extended stability tests given in Section 3 is applied to a recent version of the St. Louis equation.

2. GENERAL TEST OF EQUALITY BETWEEN FULL SETS OF COEFFICIENTS

We will find it very convenient to have some of the results given by Scheffé [1959, Sections 2.5–2.6] summarized in the following theorem which gives a geometric interpretation of likelihood ratio tests of linear hypotheses within the context of the classical linear model.

THEOREM. *Let $\underline{y}$ be a $n \times 1$ random vector following a $N[\underline{\eta}, \sigma^2 I_n]$ distribution, V_r a r-dimensional subspace of R^n and V_{r_0} a r_0-dimensional subspace of V_r, where $r_0 < r < n$. Let also*

$$S_\Omega = \min_{\underline{\eta} \in V_r} (\underline{y} - \underline{\eta})'(\underline{y} - \underline{\eta}), \qquad S_0 = \min_{\underline{\eta} \in V_{r_0}} (\underline{y} - \underline{\eta})'(\underline{y} - \underline{\eta}).$$

Then, the likelihood ratio test for testing

$$H_0 \colon \underline{\eta} \in V_{r_0}, \qquad \textit{versus} \quad \Omega \colon \underline{\eta} \in V_r,$$

is given by $\{F \geq F_\alpha\}$, where

$$F = \frac{n-r}{r-r_0} \, \frac{S_0 - S_\Omega}{S_\Omega}$$

follows an $F(r-r_0, n-r)$ distribution under the null hypothesis H_0 and F_α is the appropriately chosen critical point for a test of level α.

Furthermore, the above test can be shown to be uniformly most powerful in wide classes of alternative tests (see Scheffé [1959, pp. 46–51]), and thus the tests obtained below will enjoy the same optimal properties. Now considering equation (1.1), let us define:

(2.1) $$X^* = \begin{bmatrix} X_1 & 0 \cdots 0 \\ 0 & X_2 \cdots 0 \\ \vdots & \vdots \quad \vdots \\ 0 & 0 \cdots X_m \end{bmatrix}, \; X = \begin{bmatrix} X_1 \\ X_2 \\ \vdots \\ X_m \end{bmatrix}, \; \underline{y} = \begin{bmatrix} \underline{y}_1 \\ \underline{y}_2 \\ \vdots \\ \underline{y}_m \end{bmatrix}, \; \underline{u} = \begin{bmatrix} \underline{u}_1 \\ \underline{u}_2 \\ \vdots \\ \underline{u}_m \end{bmatrix}$$

and $r_0 = \text{rank}(X)$. The rank of X^* is thus $r = \sum_{i=1}^{m} r_i$. Under H_0, equation (1.1) can be rewritten:

(2.2) $$\underline{y} = X\underline{\beta}_0 + \underline{u},$$

where $\underline{\beta}_0 \equiv \underline{\beta}_1 = \cdots = \underline{\beta}_m$, while under the alternative,

(2.3) $$\underline{y} = X^*\underline{\beta} + \underline{u},$$

where $\underline{\beta} = (\beta'_1, \ldots, \beta'_m)'$. The subspace generated by the columns of X^* has dimension r, while the subspace generated by the columns of X is a subspace of the previous one having dimension r_0; let us call these V_r and V_{r_0} respectively. Under the null hypothesis, $E(\underline{y}) \in V_{r_0}$ while, under the alternative, $E(y) \in V_r$. Thus, by the Theorem, the likelihood ratio test for testing H_0 is given by $\{F \geq F_\alpha\}$, with

(2.4) $$F = \frac{v}{v_0} \frac{S_0 - S_\Omega}{S_\Omega},$$

where

(2.5) $$v_0 = \sum_{i=1}^{m} r_i - r_0, \qquad v = \sum_{i=1}^{m} (n_i - r_i),$$

(2.6) $$S_0 = \min_{\underline{\beta}} (\underline{y} - X\underline{\beta})'(\underline{y} - X\underline{\beta}), \qquad S_\Omega = \sum_{i=1}^{m} S_i$$

and

(2.7) $$S_i = \min_{\underline{\beta}_i} (\underline{y}_i - X_i\underline{\beta}_i)'(\underline{y}_i - X_i\underline{\beta}_i), \qquad i = 1, \ldots, m.^3$$

Under H_0, F follows a $F(v_0, v)$ distribution. Furthermore, we can see that a necessary and sufficient condition for H_0 to be testable using (2.4) is

(2.8) $$\sum_{i=1}^{m} n_i > \sum_{i=1}^{m} r_i > r_0.$$

3. SPECIAL CASES

We shall now examine important special cases of the above test, especially cases where the X_i matrices have maximum rank (either full column rank or full row rank), which are the most frequent situations in practice.

If the matrices X_i all have full column rank ($r_i = k$, $i = 1, \ldots, m$), we must also

[3] When $r_i < k$, S_i may be computed by replacing $\underline{\beta}_i$ by $\hat{\underline{\beta}}_i = (X_i'X_i)^- X_i'\underline{y}_i$, where $(X_i'X_i)^-$ is a generalized inverse of $X_i'X_i$, and similarly for S_0. See Rao [1973, Chapter 4].

have $r_0=k$, hence $v_0=(m-1)k$ and $v=\sum_{i=1}^{m} n_i-mk$ in (2.4); we thus get the same test as Kullback and Rosenblatt [1957, p. 73] for this problem. On the other hand, in the standard Chow problem, it is assumed that $m=2$, $r_1=k<n_1$ and either $r_2=k<n_2$ or $r_2=n_2\leq k$. Since $r_1=k$, we must have $r_0=k$, and thus $v_0=r_2$ and $v=n_1+n_2-k-r_2$. When $r_2=k$, we have $v_0=k$ and $v=n_1+n_2-2k$; when $r_2=n_2\leq k$ (an insufficient number of extra observations), we have $v_0=n_2$ and $v=n_1-k$. In the second case, we also have $S_2=0$, so that F takes the form:

$$F=\frac{n_1-k}{n_2}\,\frac{S_0-S_1}{S_1}, \tag{3.1}$$

which can be shown to be mathematically identical with the statistic of the predictive "Chow test" (see Chow [1960, p. 598–599]). Furthermore, it is interesting to observe that the null distributions of the two Chow tests are here obtained in a particularly simple and unified way, while this is usually done via two separate derivations.[4]

Now let us assume m_1 of the m regressions, with $0<m_1<m$, have insufficient numbers of observations ($r_i=n_i\leq k$). Call I the set of regressions such that $r_i=n_i\leq k$ and $\bar{I}=\{1,\ldots,m\}\backslash I$. Then, for each $i\in I$, we have $S_i=0$ and the statistic F takes the form:

$$F=\frac{v}{v_0}\left[(S_0-\sum_{i\in\bar{I}} S_i)/\sum_{i\in\bar{I}} S_i\right] \tag{3.2}$$

where

$$v_0=\sum_{i\in I} n_i+\sum_{i\in\bar{I}} r_i-r_0, \qquad v=\sum_{i\in\bar{I}}(n_i-r_i) \tag{3.3}$$

If, in particular, $r_i=k$ for all $i\in\bar{I}$, then $r_0=k$ and

$$v_0=\sum_{i\in I} n_i+(m_2-1)k, \qquad v=\sum_{i\in\bar{I}} n_i-m_2k. \tag{3.4}$$

where $m_2=m-m_1$. This simple generalization of the second Chow test allows us to test H_0 when more than 1 (but fewer than m) regressions have insufficient numbers of observations; one can see easily that it is not in general equivalent to the predictive test in (3.1). Furthermore, we can observe that, if the regressions included in I are not necessarily chosen such that $r_i=n_i\leq k$ (but may instead have $k<n_i$ or $r_i<n_i$, so that these regressions do not generally yield a perfect fit) and no line of X_i is zero for any $i\in I$, the statistic F as given in (3.2) will follow the same distribution $F(v_0, v)$ with v_0 and v given by (3.3). This can be seen easily by noting that each regression in I can always be divided into shorter ones (so that we have $r_i=n_i\leq k$ for each subperiod in the new subdivision) and that such a process will not change the value of F.[5]

[4] For various alternative derivations of the predictive Chow test, see Chow [1960], Fisher [1970], Harvey [1976] and Dufour [1980].

[5] A simple way to do this consists in taking subperiods of 1 observation, which makes $n_i=r_i=1$ for every subperiod in I.

4. TESTS OF EQUALITY BETWEEN SUBSETS OF COEFFICIENTS

Let us now consider the following modification of the set-up of Section 1:

$$y_i = X_{i1}\beta_{i1} + X_{i2}\beta_{i2} + u_i, \qquad i = 1,\dots, m, \tag{4.1}$$

where X_{i1} is a $n_i \times k_1$ non-stochastic matrix of rank r_{i1}, X_{i2} is a $n_i \times k_{i2}$ non-stochastic matrix of rank r_{i2}, β_{i1} and β_{i2} are $k_1 \times 1$ and $k_{i2} \times 1$ vectors of coefficients, while other assumptions remain unchanged. Note that the different regressions may have different numbers of coefficients, because the vectors β_{i2} may have different dimensions. We want to test the null hypothesis H_0': $\beta_{11} = \cdots = \beta_{m1}$.

In order to perform this test, let us define:

$$X_i = [X_{i1}, X_{i2}], \quad \beta_i = (\beta_{i1}', \beta_{i2}')', \qquad i = 1,\dots, m, \tag{4.2}$$

and

$$X_0 = \begin{bmatrix} X_{11} & X_{12} & 0 & \cdots & 0 \\ X_{21} & 0 & X_{22} & \cdots & 0 \\ \vdots & \vdots & \vdots & & \vdots \\ X_{m1} & 0 & 0 & \cdots & X_{m2} \end{bmatrix}, \quad X_{01} = \begin{bmatrix} X_{11} \\ X_{21} \\ \vdots \\ X_{m1} \end{bmatrix}. \tag{4.3}$$

Let also $r_i = \text{rank}(X_i)$, $i = 1,\dots, m$, and r_{01} = number of linearly independent columns of X_{01} which are also independent of the last $\sum_{i=1}^{m} k_{i2}$ columns of X_0.

Under the null hypothesis, equation (4.1) can be rewritten:

$$y = X_0\beta_0 + u, \tag{4.4}$$

where $\beta_0 = (\beta_{01}', \beta_{12}',\dots, \beta_{m2}')'$ and β_{01} is the common value of $\beta_{11},\dots, \beta_{m1}$, while, under the alternative,

$$y = X^*\beta + u, \tag{4.5}$$

where $\beta = (\beta_1',\dots, \beta_m')'$; y, X^* and u are defined as in (2.1). The subspace generated by the columns of X^* has dimension $r = \sum_{i=1}^{m} r_i$, while the space generated by the columns of X_0 is a subspace of the previous one having dimension $r_0' = r_{01} + \sum_{i=1}^{m} r_{i2}$; we will call these two subspaces V_r and $V_{r_0'}$ respectively. Under the null hypothesis, $E(y) \in V_{r_0'}$ while, under the alternative, $E(y) \in V_r$. Thus, by the Theorem, the likelihood ratio test for testing H_0' is given by $\{F' \geq F_\alpha\}$, with

$$F' = \frac{\nu}{\nu_0'} \frac{S_0' - S_\Omega}{S_\Omega}, \tag{4.6}$$

where

$$\nu_0' = \sum_{i=1}^{m} (r_i - r_{i2}) - r_{01}, \quad \nu = \sum_{i=1}^{m} (n_i - r_i), \tag{4.7}$$

$$S_0' = \min_{\beta_0} (y - X_0\beta_0)'(y - X_0\beta_0), \quad S_\Omega = \sum_{i=1}^{m} S_i \tag{4.8}$$

and S_i, $i=1,\ldots,m$, are defined as in (2.7). Under H_0', F' follows a $F(\nu_0', \nu)$ distribution. Furthermore, we can see that a necessary and sufficient condition for H_0' to be testable using (4.6) is

$$\sum_{i=1}^{m} n_i > \sum_{i=1}^{m} r_i > r_{01} + \sum_{i=1}^{m} r_{i2}\,. \tag{4.9}$$

We can note here that the fact that the vectors $\underline{\beta}_{i2}$, $i=1,\ldots,m$, may have different lengths allows one to exclude variables from some regressions (by setting the corresponding coefficients to zero) and, more generally, to impose various a priori restrictions on the different regressions.

If the numbers of coefficients in the m regressions are the same ($k_{i2}=k_2$, $i=1,\ldots,m$) and the matrices X_i all have full column rank ($r_i=k_1+k_2\equiv k$, $i=1,\ldots,m$), then the matrices X_{i1} and X_{i2} must also have full column rank ($r_{i1}=k_1$, $r_{i2}=k_2$, $i=1,\ldots,m$) and $r_{01}=k_1$; hence, we have $\nu_0'=(m-1)k_1$, $\nu=\sum_{i=1}^{m} n_i-mk$ in (4.6), and we get a test previously given by Kullback and Rosenblatt [1957, p. 76], and by Chow [1960, pp. 509–602] for $m=2$.

If $m=2$ and X_1 has full column rank, then $r_{11}=r_{01}=k_1$, $r_{12}=k_{12}$ and $r_1=k_1+k_{12}$. Hence, we have $\nu_0'=r_2-r_{22}$ and $\nu=n_1+n_2-k_1-k_{12}-r_2$. We can note that H_0' is not testable when $r_2=r_{22}$, and so let $r_{22}<r_2$. Furthermore, if $n_2<k_1+k_{22}$ (insufficient number of observations to run separately the second regression), if X_{22} has full column rank ($r_{22}=k_{22}$) and X_2 has full line rank ($r_2=n_2$), then we have $\nu_0'=n_2-k_{22}$, $\nu=n_1-k_1-k_{12}$, $S_2=0$ and

$$F' = \frac{n_1-k_1-k_{12}}{n_2-k_{22}}\;\frac{S_0'-S_1}{S_1}\,. \tag{4.10}$$

We thus get an extension of the test suggested by Chow [1960, p. 602], without the restriction $k_{12}=k_{22}$.

We will now extend the above test to m regressions. Assume there is a subset I containing m_1 regressions ($0<m_1<m$) such that $k_{i2}<n_i<k_1+k_{i2}$ and X_i has full column rank for every $i\in\bar{I}$. Then $r_{i1}=k_1$, $r_{i2}=k_{i2}$ and $r_i=k_1+k_{i2}$ for $i\in\bar{I}$, and $r_{01}=k_1$; hence

$$\nu_0' = (m_2-1)k_1 + \sum_{i\in I}(r_i-r_{i2}), \qquad \nu = \sum_{i=1}^{m}(n_i-r_i), \tag{4.11}$$

where $m_2=m-m_1$. If, furthermore, X_{i2} has full column rank ($r_{i2}=k_{i2}$) and X_i has full row rank ($r_i=n_i$) for every $i\in I$, then $S_i=0$ for $i\in I$, and F' takes the form:

$$F' = \frac{\nu}{\nu_0'}\left[\Big(S_0'-\sum_{i\in \bar{I}} S_i\Big)\Big/\sum_{i\in \bar{I}} S_i\right] \tag{4.12}$$

where

$$\nu_0' = (m_2-1)k_1 + \sum_{i\in I}(n_i-k_{i2}), \qquad \nu = \sum_{i\in \bar{I}}(n_i-k_1-k_{i2})\,. \tag{4.13}$$

Finally, in order to illustrate the flexibility of the above test, we shall indicate how it could be used to test a hypothesis of the type $H_0''\colon \underline{\beta}_{11}=\cdots=\underline{\beta}_{p1}$, where $0<$

$p<m$. For $i=p+1,\dots,m$, let us replace, in (4.1), X_{i2} by $\bar{X}_{i2}=X_i$, where X_i is defined by (4.2) and X_{i1} by $\bar{X}_{i1}=0_i$, a $n_i \times k_{i1}$ zero matrix; $\bar{X}_{i1}$ thus has rank zero and $\bar{X}_{i2}$ has rank r_i; then, consider the system:

$$\text{(4.14)} \qquad \begin{aligned} y_i &= X_{i1}\beta_{i1} + X_{i2}\beta_{i2} + u_i, \qquad i = 1,\dots,p, \\ y_i &= \bar{X}_{i1}\bar{\beta}_{i1} + \bar{X}_{i2}\bar{\beta}_{i2} + u_i, \qquad i = p+1,\dots,m, \end{aligned}$$

where $\bar{\beta}_{i2}=(\beta'_{i1}, \beta'_{i2})'$. It is easy to see that the null hypothesis H''_0 is here (observationally) equuivalent to H'_0: $\beta_{11}=\dots=\beta_{p1}=\beta_{(p+1)1}=\dots=\beta_{m1}$. Consequently, we can test H''_0 by testing the equivalent hypothesis H'_0, which can be done by using the procedure described in this section [provided (4.9) holds].

5. ILLUSTRATION

Of the situations covered by the above results, problems of testing stability with several subperiods, some of which are undersized (but still have maximum row rank), are likely to be the most frequent in practice. We shall illustrate now such a problem by testing the stability of a version of the "St. Louis equation" in rate-of-change form suggested by Carlson [1978]. This equation is:

$$\dot{Y}_t = \alpha + \sum_{i=0}^{4} m_i \dot{M}_{t-i} + \sum_{i=0}^{4} e_i \dot{E}_{t-i}$$

where $\dot{Y}_t$, $\dot{M}_t$ and $\dot{E}_t$ are the compounded annual rates of change in nominal GNP, money stock (M1) and high-employment expenditures respectively in the United States. The sample period considered is 1953/I–1976/IV (quarterly data). The equation was originally estimated using Almon polynomials for m_i and e_i (fourth degree polynomials constrained to go through zero at the endpoints; see Carlson [1978; Table IV]). However, we also estimated this equation without restrictions (see Table 1) and found that the F statistic for testing the Almon restrictions is quite high [$F_{4,85}=2.608>F_{.05}(4,85)$]. Consequently, we reject the Almon restrictions and we shall concentrate our analysis on the less restricted model. Besides, we may note that Carlson reports not to have found any evidence of instability of the Almon constrained version of the model after applying the Brown, Durbin and Evans [1975] techniques (though details of this analysis are not supplied by the author); of course, the instability of the unrestricted equation would imply instability of the restricted equation.

In order to test the above equation for stability over time, we divided the sample period 1953/I–1976/IV into 7 disjoint subperiods: 1953/I–1959/IV, 1960/I–1960/IV, 1961/I–1969/IV, 1970/I–1970/III, 1970/IV–1973/III, 1973/IV–1975/I and 1975/II–1976/IV. The points of divison between these periods coincide roughly with business cycle turning points (as defined by movements in real GNP). We chose this type of subdivision because we considered highly plausible the conjecture that an important structural break (e.g., the OPEC price hike in 1973/IV) would lead to (or would be associated with) a phase of recession or a phase of

TABLE 1

UNCONSTRAINED ST. LOUIS EQUATION*

$$\dot{Y}_t = \alpha + \sum_{i=0}^{4} m_i \dot{M}_{t-i} + \sum_{i=0}^{4} e_i \dot{E}_{t-i}$$

Sample period: 1953/I–1976/IV

m_0	.607 (3.277)	e_0	.055 (1.329)
m_1	.238 (1.031)	e_1	.104 (2.511)
m_2	.022 (.093)	e_2	−.0225 (−.557)
m_3	.631 (2.536)	e_3	−.0276 (−.688)
m_4	−.440 (−2.260)	e_4	−.0947 (−2.418)
Σm_i	1.059 (2.315)	Σe_i	.0141 (.211)
α	2.829 (3.507)		

$SS = 1116.54$, $R^2 = .465$, $S.E. = 3.624$, $D.W. = 1.745$, $D.F. = 85$

* t-statistics are given in parentheses, SS is the sum of squared residuals, R^2 is the coefficient of multiple determination, $S.E.$ is the standard error of the regression, $D.W.$ is the Durbin-Watson statistic and $D.F.$ is the number of degrees of freedom.

TABLE 2

UNCONSTRAINED ST. LOUIS EQUATION

SUM OF SQUARED RESIDUALS FOR SUBPERIODS

Period	Sum of squared residuals	Number of observations
1953/I - 1976/IV	1116.54	96
1953/I - 1959/IV	235.971	28
1960/I - 1960/IV	—	4
1961/I - 1969/IV	109.808	36
1970/I - 1970/III	—	3
1970/IV - 1973/III	38.7826	12
1973/IV - 1975/I	—	6
1975/II - 1976/IV	—	7

expansion. Of these subperiods, only three have a number of observations sufficient to allow a separate estimation (1953/I–1959/IV, 1961/I–1969/IV and 1970/IV–1973/III). The sums of squared residuals for the subperiods are reported in Table 2 and the general test statistic for performing the stability test is given by equations (3.2) and (3.4). Using the information in Table 2, we can see easily that $S_0 = 1116.54$, $\sum_{i \in I} S_i = 384.5616$, $\nu_0 = 42$ and $\nu = 43$. Hence we get $F = 1.949$ which is significant at a level as low as .016 (hence also at the conventional .05 level).[6] Thus, from this result, we can rather safely reject the null hypothesis that this relationship was stable over the sample period considered. Furthermore,

[6] For the Almon constrained model, the corresponding test statistic for stability is $F = 2.392$ (with $\nu_0 = 34$ and $\nu = 55$), which is (as expected) much more strongly significant (p-value$=.0019$).

we have seen that the stability test could be performed very easily despite the presence of several undersized subperiods.

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