New Keynesian Phillips Curves, structural econometrics and weak identification *

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Contents

1. New Keynesian Phillips Curves

1. For several years, macroeconomic research has been dominated by papers by either purely theoretical papers or by papers where the link with empirical data is made through calibration methods.
2. Bad news for:
   (a) econometricians;
   (b) the credibility of macroeconomics as:
      i. a scientific discipline,
      ii. an instrument of forecasting and policy analysis.
3. Calibration can help to understand the properties of a model, but it is very weak way of assessing a model,
   because it does not try to cope with blatant empirical failures in proposed models.
4. Calibration may be a useful first step in assessing models, but it is far too incomplete to be satisfactory.

5. One reason to have hope: research on New Keynesian Phillips curves,
   which has mobilized resources from econometrics, most notably
   structural econometrics,
   statistical inference in the presence of identification problems.

6. NKPC provide an interesting example where recent results on
   weal identification
   can be applied.

2. Weak identification

Several authors in the pas have noted that usual asymptotic approximations are not valid or lead to very inaccurate results when parameters of interest are close to regions where these parameters are not anymore identifiable:

Sargan (1983, Econometrica)
Phillips (1984, International Economic Review)
Phillips (1985, International Economic Review)
Gleser and Hwang (1987, Annals of Statistics)
Koschat (1987, Annals of Statistics)
Phillips (1989, Econometric Theory)
Hillier (1990, Econometrica)
Nelson and Startz (1990a, Journal of Business)
Nelson and Startz (1990b, Econometrica)
Buse (1992, Econometrica)
Maddala and Jeong (1992, Econometrica)
Choi and Phillips (1992, Journal of Econometrics)
Bound, Jaeger, and Baker (1993, NBER Discussion Paper)
Dufour and Jasiak (1993, CRDE)

Bound, Jaeger, and Baker (1995, Journal of the American Statistical Association)

McManus, Nankervis, and Savin (1994, Journal of Econometrics)

Hall, Rudebusch, and Wilcox (1996, International Economic Review)

Dufour (1997, Econometrica)

Shea (1997, Review of Economics and Statistics)

Staiger and Stock (1997, Econometrica)

Wang and Zivot (1998, Econometrica)

Zivot, Startz, and Nelson (1998, International Economic Review)

Startz, Nelson, and Zivot (1999, International Economic Review)

Perron (1999)

Stock and Wright (2000, Econometrica)

Dufour and Jasiak (2001, International Economic Review)

Dufour and Taamouti (2001)

Kleibergen (2001, 2002)

Moreira (2001, 2002)

Stock and Yogo (2002)

Stock, Wright, and Yogo (2002, Journal of Business and Economic Statistics)

Dufour (2003, Canadian Journal of Economics)

Dufour and Taamouti (2005, Econometrica)

Dufour and Taamouti (2006, Journal of Econometrics, forth.)

Surveys:

- Stock, Wright, and Yogo (2002, Journal of Business and Economic Statistics)
- Dufour (2003, Canadian Journal of Economics)

2.1. Standard simultaneous equations model

$$y = Y\beta + X_1\gamma + u \tag{2.1}$$

$$Y = X_1\Pi_1 + X_2\Pi_2 + V \tag{2.2}$$

where:

y and Y are $T\times 1$ and $T\times G$ matrices of endogenous variables,

X_i is a $T \times k_i$ matrix of exogenous variables (instruments), $i = 1,\, 2,\, 3$:

- X_1 : exogenous variables included in the structural equation;
- X_2 : exogenous variables excluded from the structural equation ;

β and γ are $G \times 1$ and $k_1 \times 1$ vectors of unknown coefficients;

Π_1 and Π_2 are $k_1 \times G$ and $k_2 \times G$ matrices of unknown coefficients;

u is a vector of structural disturbances;

V is a $T \times G$ matrix of reduced-form disturbances;

$X = [X_1, X_2]$ is a full-column rank $T \times k$ matrix, where $k = k_1 + k_2$.

This model can be rewritten in reduced form as:

$$\begin{aligned} y &= Y\beta + X_1\gamma + u \\ &= (X_1\Pi_1 + X_2\Pi_2 + V)\beta + X_1\gamma + u \\ &= X_1\pi_1 + X_2\pi_2 + v \\ Y &= X_1\Pi_1 + X_2\Pi_2 + V \end{aligned}$$

where $\pi_1 = \Pi_1\beta + \gamma$, $v = u + V\beta$, and

$$\pi_2 = \Pi_2\beta\,. \tag{2.3}$$

We want to make inference about β.
Generalization of an old problem [Fieller (1940, 1954)]: inference on the **ratio** of two parameters:

$$q = \frac{\mu_2}{\mu_1} \tag{2.4}$$

$$\mu_2 = q\mu_1 \tag{2.5}$$

.

(2.3) is the crucial equation controlling *identification* in this system: we need to be able to recuperate β from the regression coefficients π_2 and Π_2.

Rank condition for the identification of β

$$\beta \text{ is } \textbf{identifiable} \quad \text{iff} \quad \mathrm{rank}(\varPi_2) = G\,. \quad (2.6)$$

Weak instrument problem when:

1. $\mathrm{rank}(\varPi_2) < G$ (**nonidentification**)
2. or $\varPi_2$ is **close to being nonidentifiable**:
   (a) $\det(\varPi_2'\varPi_2)$ is "close to zero";
   (b) $\varPi_2'\varPi_2$ has one or several eigenvalues "close to zero".

Central problem: move from the clearly "identifiable" parameters $\varPi_2$ to the "structural parameters" β and γ in a way which remains valid even when the solution of the equation (2.3) is "ill-determined".

Incomplete model – In many situations, one would also like to allow for an alternative (incompletely specified model

$$Y = X_1 \mathit{\Pi}_1 + X_2 \mathit{\Pi}_2 + X_3 \mathit{\Pi}_3 + V \qquad (2.7)$$

where $X_3 : T \times k_3$ matrix of explanatory variables (not necessarily strictly exogenous) not used in the analysis, or more generally

$$Y = g(X_1,\ X_2,\ X_3,\ V,\ \mathit{\Pi}) \qquad (2.8)$$

2.2. Problems associated with weak identification

Weak instruments have been notorious to cause serious statistical difficulties, form the viewpoints of:

1. estimation;
2. confidence interval construction;
3. testing.

Difficulties

1. Theoretical results show that the distributions of various estimators depend in a complicated way upon unknown nuisance parameters. So they are difficult to interpret.
2. When identification conditions do not hold, standard asymptotic theory for estimators and test statistics typically collapses.
3. With weak instruments,
   (a) 2SLS becomes heavily biased (in the same direction as OLS),

(b) distribution of 2SLS is quite far the normal distribution (e.g., bimodal).

4. Standard Wald-type procedures based on asymptotic standard errors become fundamentally unreliable or very unreliable in finite samples [Dufour (1997, Econometrica)].

5. Problems were strikingly illustrated by the reconsideration by Bound, Jaeger, and Baker (1995, Journal of the American Statistical Association) of a study on returns to education by Angrist and Krueger (1991, QJE):

   - 329000 observations;
   - replacing the instruments used by Angrist and Krueger (1991, QJE) with randomly generated instruments (totally irrelevant) produced very similar point estimates and standard errors;
   - indicates that the instruments originally used were weak.

Crucial to use finite-sample approaches to produce reliable inference.

Finite-sample approaches to inference on models involving weak identification

- Dufour (1997, Econometrica)
- Dufour and Jasiak (2001, International Economic Review)
- Dufour and Taamouti (2005, Econometrica)
- Beaulieu, Dufour, and Khalaf (2005)
- Dufour and Valéry (2005)
- Dufour and Taamouti (2006, Journal of Econometrics, forth.)
- Dufour, Khalaf, and Kichian (2006a, Journal of Economic Dynamics and Control)
- Dufour, Khalaf, and Kichian (2006b)
- Dufour, Khalaf, and Kichian (2006c)

Important features

1. Procedures robust to lack of identification (or weak identification)
2. Procedures for which a finite-sample distributional theory can be supplied, at least in some reference cases
3. Limited information methods which do not require a complete formulation of the model [limited-information vs. full-information methods]
   (a) Robustness to missing instruments
   (b) Robustness to the formulation of the model for the explanatory endogenous variables

3. Weak identification and New Keynesian Phillips Curves

For basic NKPC, the issue of weak identification has been considered by several authors:

Ma (2002, Economics Letters)

Khalaf-Kichian (2004)

Mavroeidis (2004, Oxford Bulletin of Economics and Statistics)

Mavroeidis (2005, JMCB)

Yazgan-Yilmazkuday (2005, Studies in Nonlinear Dynamics and Econometrics)

Nason and Smith (2005)

Dufour, Khalaf, and Kichian (2006a, Journal of Economic Dynamics and Control)

Mavroeidis (2006)

1. Dufour, J.-M., L. Khalaf, and M. Kichian (2006a): "Inflation Dynamics and the New Keynesian Phillips Curve: An Identification Robust Econometric Analysis," *Journal of Economic Dynamics and Control*, 30 (9-10), 1707–1727.

Gali-Gertler (JME, 1999) model

$$\begin{aligned}\underbrace{\pi_t}_{\text{inflation}} &= \lambda \underbrace{s_t}_{\text{marginal costs}} + \gamma_f \boxed{E_t \pi_{t+1}} + \gamma_b \, \pi_{t-1} \\ &= \lambda s_t + \gamma_f \pi_{t+1} + \gamma_b \, \pi_{t+1} + u_{t+1}\end{aligned}$$

$$\lambda = \frac{(1-\omega)(1-\theta)(1-\beta\theta)}{\theta + \omega - \omega\theta + \omega\beta\theta}$$

$$\gamma_f = \frac{\beta\theta}{\theta + \omega - \omega\theta + \omega\beta\theta} \blacktriangleright \underline{\text{forward-looking}}$$

$$\gamma_b = \frac{\omega}{\theta + \omega - \omega\theta + \omega\beta\theta} \blacktriangleright \underline{\text{backward-looking}}$$

$$\beta \equiv \text{ subjective discount rate}$$

- Identification-robust tests and CS for model parameters $(\lambda, \gamma_f, \gamma_b)$ and (ω, θ, β) based on AR-type statistics and projection techniques.
- Rational and survey expectations studied.
- Survey expectations variants rejected.
- Model acceptable for the U.S. but not for Canada.

2. Dufour, J.-M., L. Khalaf, and M. Kichian (2006b): "Structural Estimation and Evaluation of Calvo-Style Inflation Models," Discussion paper, CIREQ, Un. de Montréal, and Bank of Canada.

Calvo-type inflation model studied by Eichenbaum and Fisher (2005) model.

3. Dufour, J.-M., L. Khalaf, and M. Kichian (2006c): "Structural Multi-Equation Macroeconomic Models: A System-Based Estimation and Evaluation Approach," Discussion paper, CIREQ, Un. de Montréal, and Bank of Canada.

Lindé (JME, 2005) multi-equation NKPC.

References

ANGRIST, J. D., AND A. B. KRUEGER (1991): "Does Compulsory School Attendance Affect Schooling and Earning?," *Quarterly Journal of Economics*, CVI, 979–1014.

BEAULIEU, M.-C., J.-M. DUFOUR, AND L. KHALAF (2005): "Testing Black's CAPM with Possibly Non-Gaussian Errors: An Exact Identification-Robust Simulation-Based Approach," Discussion paper, Centre interuniversitaire de recherche en analyse des organisations (CIRANO) and Centre interuniversitaire de recherche en économie quantitative (CIREQ), Université de Montréal.

BOUND, J., D. A. JAEGER, AND R. BAKER (1993): "The Cure can be Worse than the Disease: A Cautionary Tale Regarding Instrumental Variables," Technical Working Paper 137, National Bureau of Economic Research, Cambridge, MA.

BOUND, J., D. A. JAEGER, AND R. M. BAKER (1995): "Problems With Instrumental Variables Estimation When the Correlation Between the Instruments and the Endogenous Explanatory Variable Is Weak," *Journal of the American Statistical Association*, 90, 443–450.

BUSE, A. (1992): "The Bias of Instrumental Variables Estimators," *Econometrica*, 60, 173–180.

CHOI, I., AND P. C. B. PHILLIPS (1992): "Asymptotic and Finite Sample Distribution Theory for IV Estimators and Tests in Partially Identified Structural Equations," *Journal of Econometrics*, 51, 113–150.

DUFOUR, J.-M. (1997): "Some Impossibility Theorems in Econometrics, with Applications to Structural and Dynamic Models," *Econometrica*, 65, 1365–1389.

——— (2003): "Identification, Weak Instruments and Statistical Inference in Econometrics," *Canadian Journal of Economics*, 36(4), 767–808.

DUFOUR, J.-M., AND J. JASIAK (1993): "Finite Sample Inference Methods for Simultaneous Equations and Models with Unobserved and Generated

Regressors," Discussion paper, C.R.D.E., Université de Montréal, 38 pages.

DUFOUR, J.-M., AND J. JASIAK (2001): "Finite Sample Limited Information Inference Methods for Structural Equations and Models with Generated Regressors," *International Economic Review*, 42, 815–843.

DUFOUR, J.-M., L. KHALAF, AND M. KICHIAN (2006a): "Inflation Dynamics and the New Keynesian Phillips Curve: An Identification Robust Econometric Analysis," *Journal of Economic Dynamics and Control*, 30(9-10), 1707–1727.

——— (2006b): "Structural Estimation and Evaluation of Calvo-Style Inflation Models," Discussion paper, Centre de recherche et développement en économique (CRDE), Université de Montréal, and Centre interuniversitaire de recherche en analyse des organisations (CIRANO), Montréal, Canada.

——— (2006c): "Structural Multi-Equation Macroeconomic Models: A System-Based Estimation and Evaluation Approach," Discussion paper, Centre de recherche et développement en économique (CRDE), Université de Montréal, and Centre interuniversitaire de recherche en analyse des organisations (CIRANO), Montréal, Canada.

DUFOUR, J.-M., AND M. TAAMOUTI (2001): "Point-Optimal Instruments and Generalized Anderson-Rubin Procedures for Nonlinear Models," Discussion paper, C.R.D.E., Université de Montréal.

——— (2005): "Projection-Based Statistical Inference in Linear Structural Models with Possibly Weak Instruments," *Econometrica*, 73(4), 1351–1365.

——— (2006): "Further Results on Projection-Based Inference in IV Regressions with Weak, Collinear or Missing Instruments," *Journal of Econometrics*, forthcoming.

DUFOUR, J.-M., AND P. VALÉRY (2005): "Exact and Asymptotic Tests for Possibly Non-Regular Hypotheses on Stochastic Volatility Models," Discussion paper, Centre interuniversitaire de recherche en analyse des organisations (CIRANO) and Centre interuniversitaire de recherche en économie quantitative (CIREQ), Université de Montréal.

FIELLER, E. C. (1940): "The Biological Standardization of Insulin," *Journal of the Royal Statistical Society (Supplement)*, 7, 1–64.

——— (1954): "Some Problems in Interval Estimation," *Journal of the Royal Statistical Society, Series B*, 16, 175–185.

GLESER, L. J., AND J. T. HWANG (1987): "The Nonexistence of $100(1-\alpha)$ Confidence Sets of Finite Expected Diameter in Errors in Variables and Related Models," *The Annals of Statistics*, 15, 1351–1362.

HALL, A. R., G. D. RUDEBUSCH, AND D. W. WILCOX (1996): "Judging Instrument Relevance in Instrumental Variables Estimation," *International Economic Review*, 37, 283–298.

HILLIER, G. H. (1990): "On the Normalization of Structural Equations: Properties of Direction Estimators," *Econometrica*, 58, 1181–1194.

KLEIBERGEN, F. (2001): "Testing Subsets of Structural Coefficients in the IV Regression Model," Discussion paper, Department of Quantitative Economics, University of Amsterdam.

——— (2002): “Pivotal Statistics for Testing Structural Parameters in Instrumental Variables Regression,” *Econometrica*, 70(5), 1781–1803.

KOSCHAT, M. A. (1987): “A Characterization of the Fieller Solution,” *The Annals of Statistics*, 15, 462–468.

MA, A. (2002): “GMM Estimation of the New Phillips Curve,” *Economic Letters*, 76, 411–417.

MADDALA, G. S., AND J. JEONG (1992): “On the Exact Small Sample Distribution of the Instrumental Variable Estimator,” *Econometrica*, 60, 181–183.

MCMANUS, D. A., J. C. NANKERVIS, AND N. E. SAVIN (1994): “Multiple Optima and Asymptotic Approximations in the Partial Adjustment Model,” *Journal of Econometrics*, 62, 91–128.

MOREIRA, M. J. (2001): “Tests With Correct Size When Instruments Can Be Arbitrarily Weak,” Discussion paper, Department of Economics, Harvard University, Cambridge, Massachusetts.

——— (2002): "A Conditional Likelihood Ratio Test for Structural Models," Discussion paper, Department of Economics, Harvard University, Cambridge, Massachusetts.

NASON, J. M., AND G. W. SMITH (2005): "Identifying the New Keynesian Phillips Curve," Discussion Paper 2005-1, Federal Reserve Bank of Atlanta.

NELSON, C. R., AND R. STARTZ (1990a): "The Distribution of the Instrumental Variable Estimator and its t-ratio When the Instrument is a Poor One," *Journal of Business*, 63, 125–140.

——— (1990b): "Some Further Results on the Exact Small Properties of the Instrumental Variable Estimator," *Econometrica*, 58, 967–976.

PERRON, B. (1999): "Semi-Parametric Weak Instrument Regressions with an Application to the Risk Return Trade-Off," Discussion Paper 0199, C.R.D.E., Université de Montréal.

PHILLIPS, P. C. B. (1984): "The Exact Distribution of LIML: I," *International Economic Review*, 25, 249–261.

——— (1985): "The Exact Distribution of LIML: II," *International Economic Review*, 26, 21–36.

——— (1989): "Partially Identified Econometric Models," *Econometric Theory*, 5, 181–240.

SARGAN, J. D. (1983): "Identification and Lack of Identification," *Econometrica*, 51, 1605–1633.

SHEA, J. (1997): "Instrument Relevance in Multivariate Linear Models: A Simple Measure," *Review of Economics and Statistics*, LXXIX, 348–352.

STAIGER, D., AND J. H. STOCK (1997): "Instrumental Variables Regression with Weak Instruments," *Econometrica*, 65(3), 557–586.

STARTZ, R., C. R. NELSON, AND E. ZIVOT (1999): "Improved Inference for the Instrumental Variable Estimator," Discussion paper, Department of Economics, University of Washington.

STOCK, J. H., AND J. H. WRIGHT (2000): "GMM with Weak Identification," *Econometrica*, 68, 1097–1126.

STOCK, J. H., J. H. WRIGHT, AND M. YOGO (2002): "A Survey of Weak Instruments and Weak Identification in Generalized Method of Moments," *Journal of Business and Economic Statistics*, 20(4), 518–529.

STOCK, J. H., AND M. YOGO (2002): "Testing for Weak Instruments in Linear IV Regression," Discussion Paper 284, N.B.E.R., Harvard University, Cambridge, Massachusetts.

WANG, J., AND E. ZIVOT (1998): "Inference on Structural Parameters in Instrumental Variables Regression with Weak Instruments," *Econometrica*, 66(6), 1389–1404.

ZIVOT, E., R. STARTZ, AND C. R. NELSON (1998): "Valid Confidence Intervals and Inference in the Presence of Weak Instruments," *International Economic Review*, 39, 1119–1144.