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**TIME SERIES ANALYSIS  
EXERCISES  
STOCHASTIC PROCESSES 2**

1. Consider a  $MA(1)$  model:

$$X_t = \bar{\mu} + u_t - \theta u_{t-1}, \quad t \in \mathbb{Z}$$

where  $u_t \sim WN(0, \sigma^2)$  and  $\sigma^2 > 0$ .

- (a) Prove that the first autocorrelation of this model cannot be greater than 0.5 in absolute value.
  - (b) Find the values of the model parameters for which this upper bound is attained.
2. Let  $\{x_t : t \in \mathbb{Z}\}$  an  $MA(q)$  process. For  $q = 3, 4, 5, 6$ , check whether the following inequalities are correct:

- (a)  $|\rho(1)| \leq 0.75$  ;
- (b)  $|\rho(2)| \leq 0.90$  ;
- (c)  $|\rho(3)| \leq 0.90$  ;
- (d)  $|\rho(4)| \leq 0.90$  ;
- (e)  $|\rho(5)| \leq 0.90$  ;
- (f)  $|\rho(6)| \leq 0.90$  .

3. Consider the following models:

- (1)  $X_t = 0.5 X_{t-1} + u_t$ ,
- (2)  $X_t = 10 - 0.75 X_{t-1} + u_t$ ,
- (3)  $X_t = 10 + 0.7 X_{t-1} - 0.2 X_{t-2} + u_t$ ,
- (4)  $X_t = 10 + u_t - 0.75 u_{t-1} + 0.125 u_{t-2}$ ,
- (5)  $X_t = 0.5 X_{t-1} + u_t - 0.25 u_{t-1}$ ,
- (6)  $X_t = 0.5 X_{t-1} + u_t - 0.5 u_{t-1}$ ,

where  $\{u_t : t \in \mathbb{Z}\}$  is an *i.i.d.*  $N(0, 1)$  sequence. For each one of these models, answer the following questions.

- (a) Is this model stationary? Why?
- (b) Is this model invertible? Why?
- (c) Compute:
  - i.  $E(X_t)$ ;
  - ii.  $\gamma(k)$ ,  $k = 1, \dots, 8$ ;
  - iii.  $\rho(k)$ ,  $k = 1, 2, \dots, 8$ .
- (d) Graph  $\rho(k)$ ,  $k = 1, 2, \dots, 8$ .
- (e) Find the coefficients of  $u_t, u_{t-1}, u_{t-2}, u_{t-3}$  and  $u_{t-4}$  in the moving average representation of  $X_t$ .
- (f) Find the autocovariance generating function of  $X_t$ .
- (g) Find and graph the spectral density of  $X_t$ .
- (h) Compute the first four partial autocorrelations of  $X_t$ .