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**TIME SERIES ANALYSIS
EXERCISES
STOCHASTIC PROCESSES 1**

1. (a) Define the notion of **probability space**.
(b) Define the notion of real-valued **stochastic process** on a probability space.
2. Answer by TRUE, FALSE or UNCERTAIN to each one of the following statements. Justify briefly your answer. (Maximum: one page per question.)
 - (1) Any strictly stationary process is in L_2 .
 - (2) Any strictly stationary process is also second-order stationary.
 - (3) Any stationary process of order 3 is also stationary of order 2.
 - (4) Any asymptotically stationary process of order 3 is also asymptotically stationary process of order 2.
 - (5) A white noise is a stationary process of order 4.
3. Let $\gamma(k)$ the autocovariance function of second-order stationary process on the integers. Prove that:
 - (a) $\gamma(0) = \text{Var}(X_t)$ et $\gamma(k) = \gamma(-k)$, $\forall k \in \mathbb{Z}$;
 - (b) $|\gamma(k)| \leq \gamma(0)$, $\forall k \in \mathbb{Z}$;
 - (c) the function $\gamma(k)$ is positive semi-definite.
4. Consider a process that follows the following model:

$$X_t = \sum_{j=1}^m [A_j \cos(v_j t) + B_j \sin(v_j t)], \quad t \in \mathbb{Z},$$

where v_1, \dots, v_m are distinct constants on the interval $[0, 2\pi)$ and $A_j, B_j, j = 1, \dots, m$, are random variables in L_2 , such that

$$\begin{aligned} E(A_j) &= E(B_j) = 0, \quad E(A_j^2) = E(B_j^2) = \sigma_j^2, \quad j = 1, \dots, m, \\ E(A_j A_k) &= E(B_j B_k) = 0, \quad \text{for } j \neq k, \\ E(A_j B_k) &= 0, \quad \forall j, k. \end{aligned}$$

- (a) Show that this process is second-order stationary.
- (b) For the case where $m = 1$, show that this process is deterministic
[Hint: consider the regression of X_t on $\cos(v_1 t)$ and $\sin(v_1 t)$ based two observations.]