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TIME SERIES ANALYSIS
EXERCISES
STOCHASTIC PROCESSES 1

1. (a) Define the notion of **probability space**.
   (b) Define the notion of real-valued **stochastic process** on a probability space.

2. Answer by TRUE, FALSE or UNCERTAIN to each one of the following statements. Justify briefly your answer. (Maximum: one page per question.)
   (1) Any strictly stationary process is in L_2.
   (2) Any strictly stationary process is also second-order stationary.
   (3) Any stationary process of order 3 is also stationary of order 2.
   (4) Any asymptotically stationary process of order 3 is also asymptotically stationary process of order 2.
   (5) A white noise is a stationary process of order 4.

3. Let $\gamma(k)$ the autocovariance function of second-order stationary process on the integers. Prove that:
   (a) $\gamma(0) = Var(X_t)$ et $\gamma(k) = \gamma(-k)$, $\forall k \in \mathbb{Z}$;
   (b) $|\gamma(k)| \leq \gamma(0)$, $\forall k \in \mathbb{Z}$;
   (c) the function $\gamma(k)$ is positive semi-definite.

4. Consider a process that follows the following model:

$$X_t = \sum_{j=1}^{m} [A_j \cos(\nu_j t) + B_j \sin(\nu_j t)] \, , \; t \in \mathbb{Z} \, ,$$

   where $\nu_1, \ldots, \nu_m$ are distinct constants on the interval $[0, \, 2\pi)$ and $A_j, B_j, \; j = 1, \ldots, m$, are random variables in L_2 , such that

$$\begin{aligned} E(A_j) &= E(B_j) = 0 \, , \; E(A_j^2) = E(B_j^2) = \sigma_j^2 \, , \; j = 1, \ldots, n \, , \\ E(A_j A_k) &= E(B_j B_k) = 0, \text{ for } j \neq k \, , \\ E(A_j B_k) &= 0, \; \forall j, \, k \, . \end{aligned}$$

(a) Show that this process is second-order stationary.

(b) For the case where $m = 1$, show that this process is deterministic

[Hint: consider the regression of X_t on $\cos(\nu_1 t)$ and $\sin(\nu_1 t)$ based two observations.]