

**ECONOMETRIC THEORY**  
**EXERCICES 3**  
**INFORMATION**

1. Define the following notions:
  - (a) sufficient statistic;
  - (b) ancillary statistic;
  - (c) Fisher information;
  - (d) complete statistic.
2. Let  $Y_1, \dots, Y_n$  be independent and identically distributed random variables with the same density  $f(y; \theta)$ . Show that the order statistics are sufficient for  $\theta$ .
3. If the random variables  $Y_1, \dots, Y_n$  are independent  $N(0, \sigma^2)$ , find a sufficient statistic for  $\sigma^2$ .
4. State and demonstrate the factorization criterion for a sufficient statistic.
5. What are the sufficient statistics for an exponential model? Are these statistics minimal? Justify your answers.
6. Let  $\ell(Y; \theta)$  be the likelihood function for the sample  $Y = (Y_1, \dots, Y_n)'$ . Show that

$$I(\theta) = E \left[ - \frac{\partial^2 \log \ell(Y; \theta)}{\partial \theta \partial \theta'} \right].$$

7. When is parameter
  - (a) identifiable?
  - (b) locally identifiable?
8. When is a parametric model identifiable?