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# ADVANCED ECONOMETRIC THEORY
# EXERCISES 6

## GENERAL ISSUES IN TESTING THEORY

Reference: Gouriéroux and Monfort (1995, Chapter 14)

1. Explain the following distinctions:

   (a) significance test and specification test;

   (b) simple hypothesis and composite hypothesis;

   (c) identified test problem and non-identified test problem;

   (d) type I error and type II error.

2. We suppose that a variable $Y_t$ satisfies an equation of the form

$$Y_t = ae^{bt} + c + u_t\,, \quad t = 1, \ldots, T$$

   where $a, b, c$ are unknown parameters and $u_1, ..., u_T$ are i.i.d. $\mathrm{N}\left(0, \sigma^2\right)$ random variables.

   (a) Is this model identifiable? Why?

   (b) Decide which ones of the following hypotheses are identified:

      i. $a = 0$;
      ii. $b = 0$;
      iii. $c = 0$;
      iv. $a = b = 0$;
      v. $b = c = 0$.

   (a) When is a test preferred to another test?

   (b) Show that there does not exist an *optimal* test, except in degenerated cases.

3. Consider the experiment of tossing a coin and let:

$$Y = \begin{cases} 1, \text{ if “tail” is observed} \\ 0, \text{ if “head” is observed} \end{cases}.$$

We define $P[Y=1] = p$ and we wish to test the null hypothesis $H_0 : p \geq \frac{1}{2}$. Find all the admissible tests of $H_0$. Describe a non-admissible test.

4. **Principles for comparing tests**

   (a) Explain the difference between the Bayesian principle and Neyman's principle for selecting a test.

   (b) Explain what is a *risk diagram* for a test between two simple hypotheses.

   (c) What is a *Neyman test*?

5. State and prove the *Neyman-Pearson theorem*.

6. Consider the parametric model $(\Upsilon, (P_\theta;\, \theta \in \Theta))$ where $\Theta$ is an interval in $\mathbb{R}$.

   (a) When is the family $(P_\theta;\, \theta \in \Theta)$ a *monotone likelihood family*?

   (b) If $(P_\theta; \theta \in \Theta)$ a monotone likelihood family, give a uniformly most powerful test with level $\alpha$ for testing $H_0 : \theta \leq \theta_0$ against $H_1 : \theta > \theta_0$.

7. Consider a one-parameter exponential model with densities

$$\ell(y;\, \theta) = C(\theta)\, h(y) \exp[Q(\theta)\, T(y)]$$

   where $Q(\theta)$ is a strictly increasing function of $\theta \in \mathbb{R}$.

   (a) Find a uniformly most powerful test at level $\alpha$ for testing $H_0 : \theta \leq \theta_0$ against $H_1 : \theta > \theta_0$.

   (b) Find a locally most powerful test at level $\alpha$ for testing $H_0 : \theta \leq \theta_0$ against $H_1 : \theta > \theta_0$. [If a differentiability condition is required, explain it.]

   (c) Give a uniformly most powerful test at level $\alpha$ for testing $H_0 : \theta \leq \theta_1$ or $\theta \geq \theta_2$ against $H_1 : \theta_1 < \theta < \theta_2$, where $\theta_1 < \theta_2$.

# References

GOURIÉROUX, C., AND A. MONFORT (1995): *Statistics and Econometric Models, Volumes One and Two*. Cambridge University Press, Cambridge, U.K., Translated by Quang Vuong.