

ADVANCED ECONOMETRIC THEORY
EXERCISES 6
GENERAL ISSUES IN TESTING THEORY

Reference: Gouriéroux and Monfort (1995, Chapter 14)

1. Explain the following distinctions:
 - (a) significance test and specification test;
 - (b) simple hypothesis and composite hypothesis;
 - (c) identified test problem and non-identified test problem;
 - (d) type I error and type II error.

2. We suppose that a variable Y_t satisfies an equation of the form

$$Y_t = ae^{bt} + c + u_t, \quad t = 1, \dots, T$$

where a, b, c are unknown parameters and u_1, \dots, u_T are i.i.d. $N(0, \sigma^2)$ random variables.

- (a) Is this model identifiable? Why?
 - (b) Decide which ones of the following hypotheses are identified:
 - i. $a = 0$;
 - ii. $b = 0$;
 - iii. $c = 0$;
 - iv. $a = b = 0$;
 - v. $b = c = 0$.
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- (a) When is a test preferred to another test?
 - (b) Show that there does not exist an *optimal* test, except in degenerated cases.

3. Consider the experiment of tossing a coin and let:

$$Y = \begin{cases} 1, & \text{if "tail" is observed} \\ 0, & \text{if "head" is observed} \end{cases} .$$

We define $P[Y = 1] = p$ and we wish to test the null hypothesis $H_0 : p \geq \frac{1}{2}$. Find all the admissible tests of H_0 . Describe a non-admissible test.

4. **Principles for comparing tests**

- (a) Explain the difference between the Bayesian principle and Neyman's principle for selecting a test.
 - (b) Explain what is a *risk diagram* for a test between two simple hypotheses.
 - (c) What is a *Neyman test*?
5. State and prove the *Neyman-Pearson theorem*.

6. Consider the parametric model $(Y, (P_\theta; \theta \in \Theta))$ where Θ is an interval in \mathbb{R} .

- (a) When is the family $(P_\theta; \theta \in \Theta)$ a *monotone likelihood family*?
- (b) If $(P_\theta; \theta \in \Theta)$ a monotone likelihood family, give a uniformly most powerful test with level α for testing $H_0 : \theta \leq \theta_0$ against $H_1 : \theta > \theta_0$.

7. Consider a one-parameter exponential model with densities

$$\ell(y; \theta) = C(\theta) h(y) \exp[Q(\theta) T(y)]$$

where $Q(\theta)$ is a strictly increasing function of $\theta \in \mathbb{R}$.

- (a) Find a uniformly most powerful test at level α for testing $H_0 : \theta \leq \theta_0$ against $H_1 : \theta > \theta_0$.
- (b) Find a locally most powerful test at level α for testing $H_0 : \theta \leq \theta_0$ against $H_1 : \theta > \theta_0$. [If a differentiability condition is required, explain it.]
- (c) Give a uniformly most powerful test at level α for testing $H_0 : \theta \leq \theta_1$ or $\theta \geq \theta_2$ against $H_1 : \theta_1 < \theta < \theta_2$, where $\theta_1 < \theta_2$.

References

GOURIÉROUX, C., AND A. MONFORT (1995): *Statistics and Econometric Models, Volumes One and Two*. Cambridge University Press, Cambridge, U.K., Translated by Quang Vuong.