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Department of Economics
ECON 467
Econ 467D2: Econometrics
Final exam

No documentation allowed
Time allowed: 3 hours

40 points 1. Answer by TRUE, FALSE or UNCERTAIN to each one of the following statements, and justify briefly your answers (maximum: 1 page per statement).

(a) In the classical linear model, disturbances (errors) are uncorrelated but least squares residuals are correlated.

(b) By Studentizing least squares residuals, outliers are eliminated.

(c) The Durbin-Watson test is a test meant to detect heteroskedastic errors.

(d) The seemingly unrelated regression method is a method for correcting serial dependence in linear regressions.

(e) The generalized least squares method is a special case of the instrumental variables method.

(f) Maximum likelihood estimators can be obtained by setting the score function to zero.

(g) When specifying an ARMA model, minimizing Akaike's criterion is equivalent to minimizing the estimated standard error of the innovations of the process.

(h) The Ljung-Box statistic is always larger than the Box-Pierce statistic.

10 points 2. Consider the model described by the following assumptions:

(1) $Y_t = \sum_{j=1}^{p} \varphi_j Y_{t-j} + u_t \,, \quad t = p+1, \ldots, T \,;$

(2) $\{u_t : t = 1, \ldots, T\} \sim IID(0, \sigma^2) \,;$

(3) the polynomial $\varphi(z) = 1 - \varphi_1 z - \varphi_1 z^2 - \cdots - \varphi_p z^p$ has all its roots outside the unit circle except possibly for one which may be equal to 1.

Describe a procedure for testing the hypothesis that the polynomial $\varphi(z)$ has a root circle.

10 points 3. Let $X_1, X_2, \ldots, X_T$ be a second order stationary time series whose autocovariance function $\gamma(k)$ is known.

(a) Give the best linear forecast (in the mean square sense) of X_t given X_{t-1}.

(b) If $E(X_t) = 0$, $\gamma(k) = (.25)^k$, $k = 0, 1, 2, \dots$, and $X_3 = 2$, compute the best linear forecast of X_4.

20 points 4. Consider the linear regression model

$$y = X\beta + u \tag{1}$$

where y is a $T \times 1$ vector of observations on a dependent variable, X is a $T \times k$ nonstochastic matrix of rank k, and u is a $T \times 1$ vector of disturbances (errors) such that

$$\mathsf{E}(u) = 0\,, \tag{2}$$
$$\mathsf{V}(u) = \sigma^2 V\,, \tag{3}$$

and V is a known $T \times T$ positive definite matrix.

(a) Is the least squares estimator of β unbiased for this model? Justify your answer.

(b) Derive the best linear unbiased estimator of β for this model. How is this estimator called?

(c) Define the "weighted least squares" estimator for this model and explain why this terminology is being used.

(d) If u follows a Gaussian distribution, what is the distribution of the "weighted least squares"?

20 points 5. Consider the following demand and supply model:

$$q_t = a_1 + b_1 p_t + c_1 Y_t + u_{t1}\,, \text{ (demand function)} \tag{4}$$
$$q_t = a_2 + b_2 p_t + c_2 R_t + u_{t2}\,, \text{ (supply function)} \tag{5}$$

where

$q_t =$ quantity (at time t), $p_t =$ price, $Y_t =$ income, $R_t =$ rain volume,

u_{t1} and u_{t2} are random disturbances.

(a) Derive the reduced form of this model.

(b) Explain why applying least squares to the equations (4)-(5) may not be an appropriate method to estimate the parameters of these two equations.

(c) Are the parameters of equations (4)-(5) identified? Explain your answer.

(d) Propose an estimation method for the parameters of equations (4)-(5) and discuss its properties.