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McGill University
ECN 467
Econ 467D2: Econometrics
Mid-term exam

No documentation allowed
Time allowed: 1.5 hour

30 points 1. Consider a process that follows the following model:

$$X_t = \sum_{j=1}^{m} [A_j \cos(\nu_j t) + B_j \sin(\nu_j t)] \ , \ t \in \mathbb{Z} \ ,$$

where $\nu_1, \ldots, \nu_m$ are distinct constants on the interval $[0,\ 2\pi)$ and A_j, B_j, $j = 1,\ \ldots,\ m$, are random variables in L_2, such that

$$\begin{aligned} E(A_j) &= E(B_j) = 0 \ , \ E(A_j^2) = E(B_j^2) = \sigma_j^2 \ , \ j = 1,\ \ldots,\ n \ , \\ E(A_j A_k) &= E(B_j B_k) = 0, \ \text{for } j \neq k \ , \\ E(A_j B_k) &= 0, \ \forall j,\ k \ . \end{aligned}$$

(a) Show that this process is second-order stationary.

(b) For the case where $m = 1$, show that this process is deterministic.

(c) For $m = 1$, $\nu_1 = 1$ and $\sigma_1^2 = 10$, find the first two partial autocorrelations of X_t.

(d) For $m = 1$, $\nu_1 = 1$ $\sigma_1^2 = 10$ and $X_3 = 2$, find the best linear forecast of X_4 based on X_3.

40 points 2. Consider the following model:

$$X_t = 0.5\ X_{t-1} + u_t - 0.5\ u_{t-1} \tag{0.1}$$

where $\{u_t : t \in \mathbb{Z}\}$ is an $i.i.d.$ $N(0,1)$ sequence. Answer the following questions.

(a) Is this model stationary? Why?

(b) Is this model invertible? Why?

(c) Compute:

   i. $E(X_t)$;
   ii. $\gamma(k)$, $k = 1, \ldots, 8$;
   iii. $\rho(k)$, $k = 1, 2, \ldots, 8$.

(d) Graph $\rho(k)$, $k = 1, 2, \ldots, 8$.

(e) Find the coefficients of u_t, u_{t-1}, u_{t-2}, u_{t-3} and u_{t-4} in the moving average representation of X_t.

(f) Compute the first four partial autocorrelations of X_t.

30 points 3. Let $X_1, X_2, \ldots, X_T$ be a time series.

(a) Define the sample autocorrelations for this series.

(b) Discuss the asymptotic distributions of these two sets of autocorrelations in the following cases:

   i. under the hypothesis that $X_1, X_2, \ldots, X_T$ are independent and identically distributed (i.i.d.);
   ii. under the hypothesis that the process follows a moving average of finite order.