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**McGill University**
**Department of Economics**
**Econ 469: Econometrics**
**Final exam**

No documentation allowed
Hand calculator allowed
Time allowed: 3 hours

30 points 1. Answer by TRUE, FALSE or UNCERTAIN to each one of the following statements, and justify briefly your answers (maximum: 1 page per statement).

(a) After Studentizing least squares residuals, outliers may be difficult to detect.

(b) The Durbin-Watson test is a test meant to detect outliers.

(c) The Box-Pierce statistic is always larger than the Ljung-Box statistic.

(d) When an autoregressive model satisfies the unit root hypothesis, the associated polynomial has one root equal to one, while the other roots are smaller than one.

(e) The Dickey-Fuller procedure allows one to test whether au autoregressive model is non-stationary.

(f) The generalized least squares method is a special case of the instrumental variables method.

25 points 2. Consider the following models:

$$X_t = 10 + u_t - 0.75\, u_{t-1} + 0.125\, u_{t-2}$$

where $\{u_t : t \in \mathbb{Z}\}$ is an $i.i.d.$ $N(0,1)$ sequence. For each one of these models, answer the following questions.

(a) Is this model stationary? Why?

(b) Is this model invertible? Why?

(c) Compute:

i. $E(X_t)$ ;
ii. $\gamma(k)$ , $k = 1, \ldots , 8$;
iii. $\rho(k)$ , $k = 1, 2, \ldots , 8$.

(d) Graph $\rho(k)\,,\ k=1,\,2,\,\ldots\,,\ 8.$

(e) Find the coefficients of $u_t,\ u_{t-1},\ u_{t-2},\ u_{t-3}$ and $u_{t-4}$ in the moving average representation of $X_t$.

(f) If $X_{10}=12$ and assuming the parameters of the model are known, can you compute the best linear forecasts of $X_{10},\,X_{11},\,X_{12}$ and $X_{13}$ based on $X_{10}$ (only)? If so, compute these.

(g) If $u_{10}=2,\ u_9=1,\ u_8=0.99,\ u_7=1.2$, and assuming the parameters of the model are known, can you compute the best linear forecasts of $X_{11},\ X_{12}$ and $X_{13}$ based on the history of the process up to $X_{10}$? If so, compute these.

15 points 3. Consider the linear regression model

$$y = X\beta + u \tag{1}$$

where $y$ is a $T\times 1$ vector of observations on a dependent variable, $X$ is a $T\times k$ nonstochastic matrix of rank $k$, and $u$ is a $T\times 1$ vector of disturbances (errors) such that

$$\begin{aligned} \mathsf{E}(u) &= 0\,, & (2)\\ \mathsf{V}(u) &= \sigma^2 V\,, & (3)\end{aligned}$$

and $V$ is a known $T\times T$ positive definite matrix.

(a) Is the least squares estimator of $\beta$ unbiased for this model? Justify your answer.

(b) Derive the best linear unbiased estimator of $\beta$ for this model. How is this estimator called?

(c) Define the “weighted least squares” estimator for this model and explain why this terminology is being used.

(d) If $u$ follows a Gaussian distribution, what is the distribution of the “weighted least squares”?

10 points 4. Describe the method of seemingly unrelated regressions and explain why it may lead to efficiency improvements.

20 points 5. Consider the following demand and supply model:

$$\begin{aligned} q_t &= a_1 + b_1 p_t + c_1 Y_t + u_{t1}\,, \ \text{(demand function)} & (4)\\ q_t &= a_2 + b_2 p_t + c_2 R_t + u_{t2}\,, \ \text{(supply function)} & (5)\end{aligned}$$

where

$q_t =$ quantity (at time $t$), $p_t =$ price, $Y_t =$ income, $R_t =$ rain volume,

$u_{t1}$ and $u_{t2}$ are random disturbances.

(a) Derive the reduced form of this model.

(b) Explain why applying least squares to the equations (4)-(5) may not be an appropriate method to estimate the parameters of these two equations.

(c) Are the parameters of equations (4)-(5) identified? Explain your answer.

(d) Propose an estimation method for the parameters of equations (4)-(5) and discuss its properties.