Jean-Marie Dufour
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ECONOMETRIC THEORY
EXERCISES 1

MODELS

Reference: Gouriéroux and Monfort (1995, Chapter 1)

1. (a) Define the notion of *statistical model*.

   (b) Explain the distinction between a *dominated statistical model* and a *homogeneous statistical model*.

   (c) When is a model *nested* by another model? What is a *submodel*? What a *nesting model*?

2. (a) Explain what is an *exponential statistical model*.

   (b) Give two examples of exponential statistical models and explain why these models belong to the exponential family.

   (c) Is a linear model always an exponential model?

   (d) Which ones of the following terms apply to exponential models: parametric, nonparametric, semiparametric?

   (e) Which ones of the following terms apply to linear models: parametric, nonparametric, semiparametric?

3. Explain the difference between the Bayesian approach and the empirical Bayesian approach to the introduction of *a priori* information.

4. Let P and P^* be two probability distributions possessing densities with respect to the same measure μ.

   (a) Define the Kullback discrepancy between P and P^*.

   (b) Prove that:

      i. $I(P \mid P^*) \geq 0$;

      ii. $I(P \mid P^*) = 0 \Longleftrightarrow P = P^*$.

5. Let $y = (y_1, \ldots, y_n)'$ be a vector of observations. To explain y, we consider the linear model:
$$y = m + u\,,\ m \in L\,,\ u \sim N\left[0,\, \sigma^2 I_n\right]$$
where L is a vector $\mathbb{R}^n$ with $k.$ If the true probability distribution of y is $N\left[m_0,\, \sigma_0^2 I_n\right]$, find the pseudo true values m_0^*, σ_0^* of m and σ^2. [I_n represents the identity matrix of order n.]

6. Consider the following simple Keynesian model:
$$\begin{aligned} C_t &= aR_t + b + u_t\,, \\ Y_t &= C_t + I_t\,, \\ R_t &= Y_t\,, \end{aligned}$$
where C_t represents consumption (at time t), R_t income, Y_t production, I_t investment, and u_t is a random disturbance.

   (a) Find the reduced form of this model.

   (b) Is a coherency condition needed to derive this reduced form? If yes, which one and why?

   (c) Does this model contain *latent* variables? If so, which ones?

   (a) Explain the notion of *exogeneity* with respect to a parameter.

7. Consider the following simplified equilibrium model:
$$\begin{aligned} D_t &= \alpha + 2p_t + u_{1t}\,, \\ S_t &= c + u_{2t}\,, \\ Q_t &= D_t = S_t \qquad ,\ t = 1, \ldots, T \end{aligned}$$
where D_t is the demand for a product, S_t the supply for the same product, and Q_t the quantity produced and sold. We suppose that the vectors $(u_{1t},\ u_{2t})'$, $t = 1, \ldots, T$, are independent and $N\left[0,\ I_2\right]$.

   (a) Find the reduced form of this model.

   (b) For which parameters is the vector $Q = (Q_1, \ldots, Q_T)'$ exogenous? Justify your answer.

   (c) For which parameters is the vector $p = (p_1, \ldots, p_T)'$ exogenous? Justify your answer.

   (d) Are the variables Q_t and p_t simultaneous?

8. Prove the equivalence between non-causality in the sense of Granger and non-causality in the sense of Sims. (Define clearly these two notions.)

9. Give a sufficient condition under which *sequential exogeneity* is equivalent to *exogeneity* (for a parameter α) and justify your answer.

10. Consider the following equilibrium model:

$$\begin{aligned} &Q_t = a + bp_t + u_{1t}\,, \\ &p_t = c + dp_{t-1} + u_{2t} \qquad , \; t = 1, \ldots, T \\ &p_0 \text{ is fixed} \end{aligned}$$

where the disturbances $(u_{1t},\ u_{2t})'$, $t = 1, \ldots, T$ are independent $N[0,\ I_2]$, Q_t represents the quantity sold, and p_t the price. For which parameters is the vector $p = (p_1, \ldots, p_T)'$

   (a) sequentially exogenous?

   (b) exogenous?

   (c) strongly exogenous?

   (d) Further, does Q_t cause p_t in the sense of Granger?

   Justify your answers.

11. Consider the following equilibrium model:

$$\begin{aligned} &Q_t = a + bp_{t+1} + u_{1t}\,, \\ &p_t = c + dp_{t-1} + u_{2t} \qquad , \; t = 1, \ldots, T \\ &p_0 \text{ is fixed} \end{aligned}$$

where the disturbances $(u_{1t},\ u_{2t})'$, $t = 1, \ldots, T$ are independent $N[0,\ I_2]$, Q_t represents the quantity sold and p_t the price. For which parameters is the vector $p = (p_1, \ldots, p_T)'$

   (a) exogenous for $(a,\ b)$?

   (b) exogenous for $(c,\ d)$?

   (c) sequentially exogenous for $(a,\ b)$?

   (d) sequentially exogenous for $(c,\ d)$?

   (e) strongly exogenous for $(a,\ b)$?

(f) strongly exogenous for $(c,\ d)$?

Justify your answers.

12. Consider the following equilibrium model:

$$\begin{aligned}&Q_t = a + bp_t + u_{1t}\ ,\\ &p_t = c + dQ_{t-1} + u_{2t}\ ,\\ &Q_0 \text{ is fixed}\end{aligned}$$

where the disturbances $(u_{1t},\ u_{2t})'\ ,\ t = 1,\ \ldots\ ,\ T$ are independent $N[0,\ I_2]\ ,\ Q_t$ represents the quantity sold, and p_t the price. For which parameters is the vector $p = (p_1,\ \ldots\ ,\ p_T)'$

(a) exogenous for $(a,\ b)$?

(b) exogenous for $(c,\ d)$?

(c) sequentially exogenous for $(a,\ b)$?

(d) sequentially exogenous for $(c,\ d)$?

(e) strongly exogenous for $(a,\ b)$?

(f) strongly exogenous for $(c,\ d)$?

Justify your answers.

13. Consider the following equilibrium model:

$$\begin{aligned}&D_t = a + bp_t + u_{1t}\ ,\\ &S_t = c + dp_{t-1} + ex_t + fx_{t-1} + u_{2t}\ ,\\ &Q_t = D_t = S_t \qquad ,\ t = 1,\ \ldots\ ,\ T\end{aligned}$$

where D_t is the demand for a product, S_t the supply for the same product, Q_t the quantity produced, x_t is an exogenous variable, p_0 and x_0 are fixed, and $u_t = (u_{1t},\ u_{2t})'$ is random vector such that $E(u_t) = 0$.

(a) Give the structural form associated with this model.

(b) Give the reduced form of this model.

(c) Find the short-term multipliers for p_t and Q_t.

(d) Find the final form of the model.

(e) Find the dynamic multipliers for p_t.

(f) Find the long-run form of the model and the long-term multipliers for p_t and Q_t.

References

GOURIÉROUX, C., AND A. MONFORT (1995): *Statistics and Econometric Models, Volumes One and Two*. Cambridge University Press, Cambridge, U.K.