No documentation allowed
Time allowed: 1.5 hour

20 points 1. Answer by TRUE, FALSE or UNCERTAIN to each one of the following statements. Justify briefly your answer. (Maximum: one page per question.)

(a) If a random variable has finite second moments, it has finite moments at all higher orders.
(b) Any stationary process of order 5 is also stationary of order 2.
(c) Any strictly stationary process is in $L_2$.
(d) The Wold theorem holds for finite-order moving average processes but not autoregressive processes.
(e) Non-invertible moving processes have no covariance generating function.

20 points 2. Let $\gamma(k)$ the autocovariance function of second-order stationary process on the integers. Prove that:

(a) $\gamma(0) = Var(X_t)$ et $\gamma(k) = \gamma(-k)$, $\forall k \in \mathbb{Z}$;
(b) $|\gamma(k)| \leq \gamma(0)$, $\forall k \in \mathbb{Z}$;
(c) the function $\gamma(k)$ is positive semi-definite.

60 points 3. Consider the following models:

$$X_t = 10 + u_t - 0.75u_{t-1} + 0.125u_{t-2}, \quad (1)$$

where $\{u_t: t \in \mathbb{Z}\}$ is an i.i.d. $N(0,1)$ sequence. For each one of these models, answer the following questions.
(a) Is this model stationary? Why?
(b) Is this model invertible? Why?
(c) Compute:
   i. $E(X_t)$;
   ii. $\gamma(k), k = 1, \ldots, 8$;
   iii. $\rho(k), k = 1, 2, \ldots, 8$.
(d) Graph $\rho(k), k = 1, 2, \ldots, 8$.
(e) Find the coefficients of $u_t, u_{t-1}, u_{t-2}, u_{t-3}$ and $u_{t-4}$ in the moving average representation of $X_t$.
(f) Find the autocovariance generating function of $X_t$.
(g) Find and graph the spectral density of $X_t$.
(h) Compute the first two partial autocorrelations of $X_t$.
(i) If $X_{10} = 1$ and assuming the parameters of the model are known, can you compute the best linear forecasts of $X_{10}, X_{11}, X_{12}$ and $X_{13}$ based on $X_{10}$ (only)? If so, compute these.
(j) If $X_{10} = 1, u_{10} = 2, u_9 = 1, u_8 = 0.99, u_7 = 1.2$, and assuming the parameters of the model are known, can you compute the best linear forecasts of $X_{11}, X_{12}$ and $X_{13}$ based on the history of the process up to $X_{10}$? If so, compute these.